Rural Labour Reallocation and Productivity Growth in China\textsuperscript{1}

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Abstract

Dualism has long been a distinguished feature of many developing economies. This paper attempts to examine the possible contributions of structural change, measured by labour reallocation from agriculture to non-agriculture, to the growth of total factor productivity (TFP) in China during reform. Building upon the framework developed by Temple and WÖßmann (2006), we find that a three-sector model is more suitable to identify the role of labour reallocation to TFP growth in rural China.

JEL O11,

Keywords Labour reallocation, TFP growth, wage differentials

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1 Introduction

China had around 470 million labourers in its countryside in 2008, 50 percent more than the figure in 1980 (Table 1). Of these, 269 million were in agriculture. With just 122 million hectares of arable land (NBS, 2008), the Chinese countryside seems unable to provide enough agricultural jobs for peasants. This, together with the development of rural enterprises and other non-agricultural sectors, have attracted many agricultural labour to become engaged in all these formal and informal activities since the economic reform in the late 1970s.

Dualism has long been a distinguished feature of many developing economies. The primer works of Lewis (1954) and Ranis & Fei (1961) remain important to the basic understanding of this framework. Fei & Ranis (1997) revised and completed the whole dual economy models. The analytical framework of dualism is based on the assumption that an economy is divided into two sectors, the first being the traditional sector, and the second modern sector (A popular interpretation views the dichotomy as one between agriculture and industry). The traditional sector employs non-wage labour, or household-based decision-making units, to produce goods for subsistence purposes, while the modern sector contractually hires factor inputs to produce goods for profit maximization. In the modern sector, it is also assumed that exchange takes place through the market mechanism.

The theoretical models have been applied to study the industrialization of many countries, but surprisingly little effort has been made to China. The application of dualism to China could at least cover the following development issues:

(1) The existence of the surplus labour / inefficient labour in rural China;
(2) The transfer of such labour from the traditional sector to the modern sector (the development of rural enterprises after 1978 in particular); and
(3) The output growth contributed by such labour transfer through the growth of TFP.

This paper attempts to examine possible contributions of structural change to the growth of total factor productivity in China. To measure structural change, we focus on the reallocation of agricultural labour in the countryside to both non-agriculture in the rural and urban sectors. A three-sector model is developed to analyze the growth relationship between structural change and productivity growth.

There have been tremendous changes of GDP composition in China over the last three decades. During this period, the structural change of labour market contributed positively to the employment creation. Figure 1 shows that labour participated in agriculture dropped from nearly 60 percent in 1980 to
around 35 percent of total employment in 2008. Urban employment, on the other hand, shows steady growth from some 25 percent to almost 40 percent over the same period. More importantly, labour employment in rural non-agriculture (represented by rural enterprise workers in the early years and employment from various non-state sectors in more recent years) expanded from less than 10 percent in 1980 to more than 25 percent in 2008.

The rest of the paper is organized as follows. Section 2 discusses the development of labour market of China. Brief literature of growth related to structural change is reviewed in section 3. We derive our three-sector framework in section 4 and present the empirical model in section 5. The initial results are discussed in sector 6. Section 7 summaries our major findings.

Figure 1: Distribution of Labour Employment in China

Source: NBS.

2 Rural Labour Market of China

Rural China appears to fit the dualistic framework reasonably well (Putterman 1992). Before 1978, the collectivized institutional framework of agriculture, mainly represented by the rural communes and the household registration system (hukou), was an effective mechanism for controlling the huge rural population, in accordance with the strategic imperative of prioritizing heavy industrial development. However, with the modification of this strategy when the economic reform started at the end of the 1970s, rural enterprises (mainly industrial firms) became the main non-agricultural entities that “absorbed” rural labour. Since then, those who stayed in farming had been more successful in engaging in farming activities under various forms of the household
responsibility system, especially the popular Baogan Daohu (contracting everything to the households).

### Table 1: Rural Labour & Its Composition in China (1980-2008, Selected Years)

<table>
<thead>
<tr>
<th>Year</th>
<th>Rural L</th>
<th>RE L</th>
<th>PE L</th>
<th>Ag L</th>
<th>Rural L</th>
<th>RE L</th>
<th>PE L</th>
<th>Ag L</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td>1980</td>
<td>318.4</td>
<td>30.0</td>
<td>0.0</td>
<td>298.1</td>
<td>9.4</td>
<td>0.0</td>
<td>93.6</td>
<td>3.0</td>
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<td>1985</td>
<td>370.7</td>
<td>69.8</td>
<td>0.0</td>
<td>303.5</td>
<td>18.8</td>
<td>0.0</td>
<td>81.9</td>
<td>5.1</td>
</tr>
<tr>
<td>1990</td>
<td>477.1</td>
<td>92.7</td>
<td>16.0</td>
<td>368.4</td>
<td>19.4</td>
<td>3.4</td>
<td>77.2</td>
<td>4.1</td>
</tr>
<tr>
<td>1995</td>
<td>490.3</td>
<td>128.6</td>
<td>35.3</td>
<td>326.4</td>
<td>26.2</td>
<td>7.2</td>
<td>66.6</td>
<td>5.3</td>
</tr>
<tr>
<td>2000</td>
<td>489.3</td>
<td>128.2</td>
<td>40.7</td>
<td>320.4</td>
<td>26.2</td>
<td>8.3</td>
<td>65.5</td>
<td>5.4</td>
</tr>
<tr>
<td>2005</td>
<td>484.9</td>
<td>142.7</td>
<td>44.9</td>
<td>297.3</td>
<td>29.4</td>
<td>9.3</td>
<td>61.3</td>
<td>6.1</td>
</tr>
<tr>
<td>2007</td>
<td>476.4</td>
<td>150.9</td>
<td>48.6</td>
<td>276.9</td>
<td>31.7</td>
<td>10.2</td>
<td>58.1</td>
<td>6.6</td>
</tr>
<tr>
<td>2008</td>
<td>472.7</td>
<td>154.5</td>
<td>49.5</td>
<td>268.7</td>
<td>32.7</td>
<td>10.5</td>
<td>56.8</td>
<td>6.9</td>
</tr>
</tbody>
</table>

(2) – (5) are in million labourers; (6) – (9) are %.


### 3 Brief Literature Review

There are various theoretical approaches of identifying sources of output growth. The Solow neoclassical growth framework is considered the first structural growth model. Mankiw et al (1992) employed in the growth equation exogenous technology and diminishing returns to capital which provided good explanation of output differences among countries. These models were later extended to include human capital as input to the production for innovations, the so-called endogenous growth models (such as Howitt, 2000). Nevertheless, these models are criticized mainly by ignoring some other important variables.

The second approach is to use ad hoc regressions to incorporate all relevant variables, a Barro-type regressions after Barro (1991). Such informal regressions are popular because they can include important factors other than conventional inputs. These reduced-form growth regressions are subject to the problem of model uncertainty.

One of the criticisms of the growth accounting exercise is treating TFP as a residual, which include factors like structural change, improvement in allocative efficiency, economies of scale, and other omitted variables. The third type of model intends to rectify this shortcoming by concentrating on the role of technological efficiency in determining economic growth.

Studies of economic growth or dual economy models are often criticized for ignoring the role of structural change. As confirmed by many empirical
studies, there exists a significant differential of marginal product of labour across sectors in developing economies. Changes in the composition of employment should be, therefore, considered as an independent factor of accounting for source of economic growth.

Temple and WÖßmann (2006) attempt to incorporate structural change terms into the augmented Solow growth model so as to capture the role of both factor accumulation and productivity growth in variations on output growth. In addition to the standard determinants of aggregate TFP, Temple and WÖßmann include two approximations in their empirical models to measure the structural change. The first is approximated by the growth of non-agricultural employment and the second is the expression of migration propensity times the change of non-agricultural employment. This model exhibits several advantages. Firstly, it allows the effect of labour reallocation between sectors with different productivity into productivity growth. Secondly, no capital stock measurement is required in this model. Finally, this structural growth model is less subjective to the problem of model uncertainty than the Barro-type ad hoc model.

Following this, Ding and Knight (2009) applies this framework to a cross-country panel data analysis with system GMM estimates which includes China. They found that this extended augmented Solow model provides a good explanation of China’s output growth: actual annual growth of per worker GDP at 7.2% falls with the 95% confidence interval for its predicted value (6.3%). The unexplained residual might represent China’s efficiency gains from reform and marketization. However, the main limitation of using this framework is its failure to take proper account of labor movement with the rural sector. We aim to address this problem in our model developed below.

4 The Theoretical Framework

In this paper, we develop a three-sector model to capture the effect of structural change in China. Instead of the division between rural and urban sectors that often used in other papers, we adopt a different framework by segmenting the Chinese economy into three sectors: rural agriculture, rural non-agriculture and urban sectors, respectively. There are several advantages of such division. Firstly, the non-farm agricultural sectors enjoyed a profound development after the agricultural reform in the end of the 1970s, and therefore labour productivity in Chinese agricultural sector is expected to change significantly. Second, a lot of peasants in rural China have been employed by different non-agricultural sectors since reform, initially by the rural enterprises (mainly the township and village enterprises), and later formal and informal productions in the countryside. Due to the job nature, marginal product of labour in agriculture and that in rural non-agricultural are expected to be significantly different. Thirdly, some of the rural labour is believed to work in various urban
formal and informal productions in the presence of the household registration system. With China’s WTO membership in 2001, labour participate in the urban sector is even more obvious. The work patterns are thus vary across sectors.

The direction of labour reallocation in our three-sector model is more complicated than that under the rural-urban migration. To simplify our analysis, we only consider the possibility of people moving from agriculture to rural non-agriculture or from agriculture to urban sector.

We first define the shares of labour in the three sectors \( a_a, a_b, a_m \) as

\[
a_a = \frac{l_a}{L}; \quad a_b = \frac{l_b}{L}; \quad a_m = \frac{l_m}{L}
\]  

[1]

where the subscripts \( a, b, m \) denote rural agriculture, rural non-agriculture, and urban sectors respectively.

The extents of structural change in rural non-agricultural sector \( (P_1) \) and urban sector \( (P_2) \) are measured by

\[
P_1 = \frac{\dot{a}_b}{a_b}
\]  

[2]

\[
P_2 = \frac{\dot{a}_m}{a_m}
\]  

[3]

The dots refer to the derivatives of shares of labour with respect to time. The \( P \)s are alternatively interpreted as propensity to migrate.

In the long-run, it is assumed that no labour reallocation due to wage differentials will happen. Accordingly, long-run wage \( \bar{W} \)'s in the three sectors have the relationships as follows.

\[
\bar{W}_b = \kappa_1 \bar{W}_a \quad \kappa_1 \geq 1
\]  

[4]

\[
\bar{W}_m = \kappa_2 \bar{W}_a \quad \kappa_2 \geq 1
\]  

[5]

and the parameter \( \kappa \)'s measure the intersectoral wage ratio when labour reallocation / migration does not occur in the long-run.

The decision to work in other sectors of individual labour is based on the long-run and short-run wage ratio differentials. Since the scenario when wage rates equal to marginal products of labour is hardly observable, a simplified assumption is chosen to model such relationship. According to Temple and Wölßmann (2006), “we can use the observed extend of structural change to infer the magnitude of the wage differential”, and only the current ratio of wages
between sectors is considered. The propensity to migrate from rural agriculture to rural non-agriculture is specified as this form:

\[ P_1 = \frac{x_1}{1+x_1} \quad [6] \]

and

\[ X_1 = \varphi_1 \left( \frac{W_b}{W_a} - 1 \right) \quad [7] \]

where \( W \) denotes the wage in short-run and \( \varphi \) captures the speed of adjustment from short-run to long-run equilibrium. \( X_1 \) indicates the difference between the short-run and long-run wage ratio where the latter is 1. We assume \( W_b > W_a \) and therefore labour reallocation could occur in the short-run. Moreover, the relationship in [6] is consistent with the fact that the propensity to migrate will decrease as the agricultural wage increases.

Similarly, labour reallocation from rural agriculture to urban sector has the following relationship:

\[ P_2 = \frac{x_2}{1+x_2} \quad [8] \]

\[ X_2 = \varphi_2 \left( \frac{W_m}{W_a} - 1 \right) \quad [9] \]

The unobservable wage rates are expressed in terms of propensity to migrate and other parameters:

\[ \frac{W_b}{W_a} = \kappa_1 \left( 1 + \frac{1}{\varphi_1} \frac{P_1}{1-P_1} \right) \quad [10] \]

\[ \frac{W_m}{W_a} = \kappa_2 \left( 1 + \frac{1}{\varphi_2} \frac{P_2}{1-P_2} \right) \quad [11] \]

The total production is the sum of output in three sectors:

\[ Y = Y_a + q_b Y_b + q_m Y_m \quad [12] \]

where \( q_b \) and \( q_m \) are relative prices.

Given the factor inputs capital \( K_i \), labour \( L_i \), land \( N_i \) (fixed in our case) and technology \( A_i \), the sectoral production functions are:

\[ Y_a = A_a F(K_a, L_a, N_a) \quad [13] \]

\[ Y_b = A_b G(K_b, L_b, N_b) \quad [14] \]

\[ Y_m = A_m H(K_m, L_m, N_m) \quad [15] \]
If the labour markets in each sector are competitive, workers are assumed to be paid their marginal products:

\[ W_a = A_a F_L \]  \[ W_b = q_b A_b G_L \]  \[ W_m = q_m A_m H_L \]

where the subscript \( L \) refers to the derivative of production function with respect to labour.

The capital markets are assumed to be competitive and the rentals are constant across the three sectors:

\[ A_a F_K = q_b A_b G_K = q_m A_m H_K = r \]

where \( r \) is the rent and the subscript \( K \) refers to the derivative of the production function with respect to capital.

When the sectoral production functions are homogenous of degree one, the labour share \( \eta \) and the capital share \( 1 - \eta \) are expressed as follows:

\[ \eta = \frac{W_a L_a + W_b L_b + W_m L_m}{Y} \]  \[ 1 - \eta = \frac{r_K}{Y} \]

Take total differentiation of Eq [12] with respect to time, we have

\[ \frac{\dot{Y}}{Y} = \frac{\dot{Y}_a}{Y} + \frac{q_b \dot{Y}_b}{Y} + \frac{q_m \dot{Y}_m}{Y} \]

Alternatively the aggregate output growth is written in terms of the share of each sector:

\[ \frac{\dot{Y}}{Y} = (1 - S_b - S_m) \frac{\dot{Y}_a}{Y} + S_b \frac{\dot{Y}_b}{Y} + S_m \frac{\dot{Y}_m}{Y} \]

where

\[ S_b = \frac{q_b \dot{Y}_b}{Y_a + q_b \dot{Y}_b + q_m \dot{Y}_m} \]  \[ S_m = \frac{q_m \dot{Y}_m}{Y_a + q_b \dot{Y}_b + q_m \dot{Y}_m} \]
With the standard Cobb-Douglas technology, the aggregate production function can be written in the form of

\[
\frac{\dot{y}}{y} = \frac{\dot{z}}{z} + \eta \frac{\dot{l}}{L} + (1 - \eta) \frac{\dot{k}}{K} \tag{26}
\]

where \(Z\) denotes the aggregate technology and the \(\frac{\dot{z}}{z}\) is the growth of total factor productivity. Since our model has three sectors, Eqs. [22] and [26] could be treated as the starting point of our empirical work. Let us rewrite Eq [26] as:

\[
\frac{\dot{z}}{z} = \frac{\dot{y}}{y} - \eta \frac{\dot{l}}{L} - (1 - \eta) \frac{\dot{k}}{K} \tag{27}
\]

With Eqs [10] and [11], Eq [21] could be expressed in the following:

\[
\eta = \frac{W_a A_a L + W_a k_a (1 + \frac{p_1}{\varphi_1 P_1}) a_a L + W_a k_m (1 + \frac{p_2}{\varphi_2 P_2}) a_m L}{Y} \tag{28}
\]

Together with Eqs. [22] and [28], Eq. [27] is further expanded into:

\[
\frac{\dot{Z}}{Z} = (1 - S_b - S_m) \frac{\dot{A}_a}{A_a} + S_b \frac{\dot{A}_b}{A_b} + S_m \frac{\dot{A}_m}{A_m} + \phi(k_1 - 1) \dot{a}_b + \phi(k_2 - 1) \dot{a}_m
\]

\[+ \phi k_1 \frac{1}{\varphi_1 - 1} \left( \frac{p_1}{\varphi_1} \right) \dot{a}_b + \phi k_2 \frac{1}{\varphi_2 - 1} \left( \frac{p_2}{\varphi_2} \right) \dot{a}_m \tag{29}\]

Finally the relationship between productivity growth and structural change of labour employment is

\[
\frac{\dot{z}}{Z} = (1 - S_b - S_m) \frac{\dot{A}_a}{A_a} + S_b \frac{\dot{A}_b}{A_b} + S_m \frac{\dot{A}_m}{A_m} + \phi(k_1 - 1) GROWTH_b + \phi(k_2 - 1) GROWTH_m + \phi k_1 \varphi_1 DISEQ_b + \phi k_2 \varphi_2 DISEQ_m \tag{30}\]

Eq [30] is proved in Appendix A. Accordingly, the structural change is measured by the terms \(GROWTH_b\), \(GROWTH_m\), \(DISEQ_b\), and \(DISEQ_m\).

Intuitively, the aggregate TFP is decomposed into three parts. The first part is the weighted sum of sectoral TFP growth. The second part is the labour growth of rural non-agriculture and urban sector. The third part is the change of disequilibrium of labour shares. If we consider the case of a two-sector model, i.e., between agriculture and urban sectors, all the terms with subscript \(b\) will be removed. That case, Eq [30] will collapse to the dual economy models developed by Temple and WÖßmann (2006).
5 The Empirical Model

We derive a consistent empirical model to estimate Eq [30] with the following aggregate production function:

\[ Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \]  \[31\]

Let us denote \( k = K/AL \) and \( y = y/AL \) and assume \( A_t = A(0)e^{gt} \) and \( L_t = L(0)e^{nt} \), where \( g \) is the growth rate of technology and \( n \) is the growth rate of labour. So at the steady state can be shown that

\[ \ln \frac{Y_t}{L_t} - \ln \frac{Y(0)}{L(0)} = \theta \ln A(0) + gt + \frac{\theta a}{1-\alpha} \ln s - \frac{\theta a}{1-\alpha} \ln (n + g + \delta) - \theta \ln \frac{Y(0)}{L(0)} [32] \]

which is proved in the Appendix B. Eq [32] implies that the economy will converge to the state steady from the initial year. When there is no structural change, the TFP growth in the Solow model is expressed as:

\[ \frac{\dot{z}}{z} = \frac{\dot{A}}{A} = (1 - S_b - S_m) \frac{\dot{A}_a}{A_a} + S_b \frac{\dot{A}_b}{A_b} + S_m \frac{\dot{A}_m}{A_m} = g \]  \[33\]

In the MRW model developed by Mankiw et.al. (1992), the TFP growth rate \( g \) is assumed constant. After introducing the structural change, the TFP growth rate will be a function of several variables. Assuming \( \ln A(0) \) and \( \ln s \) are constant across provinces, our empirical model for cross-section regression is

\[ \ln \frac{Y_t}{L_t} - \ln \frac{Y(0)}{L(0)} = \text{constant} - \frac{\theta_a}{1-\alpha} \ln (n + g + \delta) - \theta \ln \frac{Y(0)}{L(0)} + \frac{t \phi(k_1-1)}{1-\alpha} GROWTH_b + \frac{t \phi(k_2-1)}{1-\alpha} GROWTH_m + \frac{t \phi_k}{\phi_2(1-\alpha)} DISSEQ_b + \frac{t \phi_k}{\phi_3(1-\alpha)} DISSEQ_m \]  \[34\]

6 Initial Results

We begin by estimating the model of Temple and WOßmann for 1983-2006. The dependent variable is the natural logarithmic (ln) difference of per capita GDP between 1983 and 2008. The standard explanatory variables include ln of the sum of labour growth rate, production growth and deprecation (which is assumed 5% in our case); and the ln of initial per capita income. The additional independent variables are the two structural change terms introduced by Temple and WOßmann (2006), as defined in Eqs (A16) – (A19). All observations are from 29 provinces for 1983 and 2008. Both the two-sector and the three-sector models are first estimated by least squares regression, and further re-estimated after adjusting heteroscedasticity.
The results of two-sector model by Temple and WÖßmann (2006) are presented in the last three columns of Table 2. Only the two structural change terms of the urban sectors are included. It is shown that both the $\ln(n + g + \delta)$ and the $DISEQ_m$ are statistically insignificant.

We then estimate our three-sector models. With Model I, all estimated coefficients are to our expected sign. Same for P-values except those for $\ln(n + g + \delta)$ and $DISEQ_0$. Model II supplements the Model I by dropping the $DISEQ$ and we find that the explanatory power of the model is very close to Model One.

Overall, the adjusted $R^2$ are very high in both models: more than 80 percent of the log difference of GDP per capita over the specified time period is explained by this three-sector model. The three-sector framework is 19 percent better than the Temple and WÖßmann (2006)’s two-sector specification (in terms of adjusted $R^2$). Such initial empirical results strengthen our theoretical specification in the case of China’s labour reallocation experience.
Table 2: Effects of Structural Change on TFP Growth

<table>
<thead>
<tr>
<th>Variable</th>
<th>Three-Sector Model</th>
<th>Two-Sector Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model I</td>
<td>Model II</td>
</tr>
<tr>
<td>ln(n + g + δ)</td>
<td>0.03</td>
<td>-1.01</td>
</tr>
<tr>
<td>ln(Y(0)/L(0))</td>
<td>-0.48</td>
<td>-0.51</td>
</tr>
<tr>
<td>GROWTH_b</td>
<td>2.71</td>
<td>1.97</td>
</tr>
<tr>
<td>GROWTH_m</td>
<td>1.66</td>
<td>2.09</td>
</tr>
<tr>
<td>DISEQ_b</td>
<td>-0.18</td>
<td>-</td>
</tr>
<tr>
<td>DISEQ_m</td>
<td>0.06</td>
<td>-0.03</td>
</tr>
<tr>
<td>Constant</td>
<td>1.45</td>
<td>1.40</td>
</tr>
<tr>
<td>R^2</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.85</td>
<td>0.83</td>
</tr>
</tbody>
</table>

The white standard errors are presented. Dependent variable is the natural log difference of per capita GDP over 2008 and 1983. The Two-Sector Model was developed by Temple and WÖßmann (2006).
Modeling structural change, measured by the reallocation of labour employment, to the contribution of TFP growth is both theoretically and empirically difficult for an underdeveloped dualistic economy. Temple and WÖßmann (2006) developed a two-sector model and found that structural change can account for a significant proportion of any observed variations in productivity growth.

Building upon their models, we extended the framework to better fit the reality of China which experienced both rural industrialization and urban industrialization during the reform period. Since there are significant differentials of marginal product of labour in rural agriculture, rural non-agricultural sectors and urban sectors, we propose a three-sector model and examine the contribution of labour reallocation to the growth of total factor productivity. Our initial results show that the specification of three-sector model is empirically superior to the framework proposed by Temple and WÖßmann (2006) at least in the case of China during reform.

References


Appendix A

By taking the derivatives of each sectoral production functions with respect to time, we obtain

\[
\begin{align*}
\frac{\dot{y}_a}{y_a} &= \frac{\dot{A}_a}{A_a} \left( \frac{A_aF(.)}{y_a} \right) + \frac{A_aF_KK}{y_a} \left( \frac{\dot{K}_a}{K} \right) + \frac{A_aF_L}{y_a} \left( \frac{\dot{L}_a}{L} \right) \tag{A.1} \\
\frac{\dot{y}_b}{y_b} &= \frac{\dot{A}_b}{A_b} \left( \frac{A_bG(.)}{y_b} \right) + \frac{A_bG_KK}{y_b} \left( \frac{\dot{K}_b}{K} \right) + \frac{A_bG_L}{y_b} \left( \frac{\dot{L}_b}{L} \right) \tag{A.2} \\
\frac{\dot{y}_m}{y_m} &= \frac{\dot{A}_m}{A_m} \left( \frac{A_mH(.)}{y_m} \right) + \frac{A_mC_K}{y_m} \left( \frac{\dot{K}_m}{K} \right) + \frac{A_mC_L}{y_m} \left( \frac{\dot{L}_m}{L} \right) \tag{A.3}
\end{align*}
\]

Multiply both sides of [A.1] to [A.3] by \((1 - S_b - S_m)\), \(S_b\) and \(S_m\), consequently,

\[
\begin{align*}
(1 - S_b - S_m) \frac{\dot{y}_a}{y_a} &= (1 - S_b - S_m) \frac{\dot{A}_a}{A_a} + (1 - \eta) \left( \frac{\dot{K}_a}{K} \right) + \frac{W_a}{Y} \left( \frac{\dot{L}_a}{L} \right) \tag{A.4} \\
S_b \frac{\dot{y}_b}{y_b} &= S_b \frac{\dot{A}_b}{A_a} + (1 - \eta) \left( \frac{\dot{K}_b}{K} \right) + \frac{W_b}{Y} \left( \frac{\dot{L}_b}{L} \right) \tag{A.5} \\
S_m \frac{\dot{y}_m}{y_m} &= S_m \frac{\dot{A}_m}{A_m} + (1 - \eta) \left( \frac{\dot{K}_m}{K} \right) + \frac{W_m}{Y} \left( \frac{\dot{L}_m}{L} \right) \tag{A.6}
\end{align*}
\]

The factor share of labour in Eq [20] is further extended to

\[
\eta = \frac{W_a\alpha a}{Y} \left[ W_a\kappa_1 \left( 1 + \frac{1}{\varphi_1 - P_1} \right) \alpha aL \right] + \left[ W_a\kappa_2 \left( 1 + \frac{1}{\varphi_2 - P_2} \right) \alpha aL \right] \tag{A.7}
\]

Define

\[
\phi = \frac{W_a\ell}{Y} \tag{A.8}
\]

With Eqs [10] and [11], factor shares of labour in sector \(b\) and \(m\) become:

\[
\frac{W_b\ell}{Y} = \phi \kappa_1 \left( 1 + \frac{1}{\varphi_1 - P_1} \right) \tag{A.9}
\]
Adding up Eqs [A.4], [A.5] and [A.6]:

\[
\frac{\dot{y}}{y} = (1 - \eta) \frac{\dot{K}}{K} + (1 - S_b - S_m) \frac{\dot{A}_a}{A_a} + S_b \frac{\dot{A}_b}{A_b} + S_m \frac{\dot{A}_m}{A_m} + \phi \frac{\dot{L}_a}{L} + \\
\phi \kappa_1 \left( 1 + \frac{1}{\varphi_1} \frac{P_1}{1 - P_1} \right) \frac{\dot{L}_b}{L} + \phi \kappa_2 \left( 1 + \frac{1}{\varphi_2} \frac{P_2}{1 - P_2} \right) \frac{\dot{L}_m}{L}
\]

[A.11]

With Eq [1], \( \frac{i_a}{l} = \alpha_a \frac{i_a}{l_a} \), \( \frac{i_b}{l} = \alpha_b \frac{i_b}{l_b} \), \( \frac{i_m}{l_m} = \alpha_m \frac{i_m}{l_m} \), Eq [A.11] becomes

\[
\frac{\dot{y}}{y} = (1 - \eta) \frac{\dot{K}}{K} + (1 - S_b - S_m) \frac{\dot{A}_a}{A_a} + S_b \frac{\dot{A}_b}{A_b} + S_m \frac{\dot{A}_m}{A_m} + \phi \alpha_a \left( \frac{l_a}{l_a} - \frac{i_a}{l} \right) + \\
\phi \kappa_1 \left( 1 + \frac{1}{\varphi_1} \frac{P_1}{1 - P_1} \right) \alpha_b \left( \frac{i_b}{L_b} - \frac{i_a}{L} \right) + \phi \kappa_2 \left( 1 + \frac{1}{\varphi_2} \frac{P_2}{1 - P_2} \right) \alpha_m \left( \frac{i_m}{L_m} - \frac{i_a}{L} \right)
\]

[A.12]

By taking the derivatives of both sides of \( L_a = a_a L \) yields \( \dot{L}_a = \dot{a}_a L + a_a \dot{L} \).

Accordingly,

\[
\frac{a_a}{\dot{a}_a} = \frac{l_a}{L_a} - \frac{L}{L}
\]

[A.13]

The same can be applied to sectors \( b \) and \( m \). Eq [A.12] is changed to

\[
\frac{\dot{y}}{y} = (1 - \eta) \frac{\dot{K}}{K} + (1 - S_b - S_m) \frac{\dot{A}_a}{A_a} + S_b \frac{\dot{A}_b}{A_b} + S_m \frac{\dot{A}_m}{A_m} + \phi \alpha_a \left( 1 + 1\varphi 1P11-P1ab+\phi \kappa 21+1\varphi 2P21-P2am \right) \]

[A.14]

Since \( a_a + a_b + a_m = 1 \), we get \( -\dot{a}_a = \dot{a}_b + \dot{a}_m \) and

\[
\frac{\dot{y}}{y} = (1 - \eta) \frac{\dot{K}}{K} + (1 - S_b - S_m) \frac{\dot{A}_a}{A_a} + S_b \frac{\dot{A}_b}{A_b} + S_m \frac{\dot{A}_m}{A_m} + \phi(k_1 - 1) \dot{a}_b \\
+ \phi(k_2 - 1) \dot{a}_m + \phi \kappa_1 \left( 1 + \frac{1}{\varphi_1} \frac{P_1}{1 - P_1} \right) \dot{a}_b + \phi \kappa_2 \left( 1 + \frac{1}{\varphi_2} \frac{P_2}{1 - P_2} \right) \dot{a}_m
\]

[A.15]

Denote
Together with Eq [26], we demonstrate the relationship between productivity growth and structural change of employment in Eq [30].

Appendix B

The capital stock evolves according to

\[ k_t = (1 - \delta)k_t + I_t \]  

[B1]

where \( \delta \) is the depreciation rate and \( I \) is the investment. In the steady state, the value of capital stock is

\[ k^* = \left[ \frac{s}{(n + g + \delta)} \right]^{\alpha/(1 - \alpha)} \]  

[B2]

Substituting the value of capital stock in the steady state into the production function yields

\[ y^* = \left[ \frac{s}{(n + g + \delta)} \right]^{\alpha/(1 - \alpha)} \]  

[B3]

The speed of convergent is given by

\[ \frac{d \ln y_t}{dt} = \lambda (\ln y^* - \ln y_t), \]

where \( \lambda = (n + g + \delta)(1 - \alpha) \)  

[B4]

Solving the above differential equation yields

\[ \ln y_t = (1 - e^{-\lambda t}) \ln y^* + e^{-\lambda t} \ln y(0) \]  

[B5]
With $y'$ in Eq [B1], denoting $\theta = 1 - e^{-\lambda t}$ and subtracting $\ln y(0)$ of both sides give Eq [34].