A Paradox of the Theory of Comparative Advantage

Baomin Dong*and Longfang Yan

November 26, 2009

Abstract

We study the gain from trade in Heckscher-Ohlin framework when a country intentionally divides itself into two regions with different factor ratios in order to gain more from trade. It is shown that a country may obtain additional gain from not specializing on the sector with comparative advantage. It is even more surprising that world welfare may be higher under such anti-comparative advantage domestic regionalization. We then claim that this is a paradox. And we agrue that the improvement in terms of trade explains the abnomality. The model can also provide a justification for "self-sufficiency" programs or "import substitution" industries in some developing countries.

Keywords: Regionalization, Heckscher-Ohlin, Comparative Advantage, Paradox

JEL classification: F10, F11

*Corresponding author. Department of Economics, University of International Business and Economics, Beijing, China, 100029, Email: baomindong@uibe.edu.cn

1. Introduction

There are many challenges to the comparative advantage theory and consequent free trade claim. Most economists firmly believe David Ricardo's celebrated comparative advantage theory in which free trade always benefits both trading countries and countries should specialize in sectors which use factors that relatively intensively endowed in the country of interest. The theory receives many criticisms.

Much of the critics we heard from the media or the people around us are fallacious. There are many intellectuals, mostly from non-economists, who value ideas and find many other economic concepts or ideas acceptable but balk at the concept of comparative advantage. There are journalists who have the reputation of deep thinkers refuse to accept the concept. There are also policy wonks who happily watch hundreds of hours of talking heads droning on about the global economy refuse to sit still for the ten minutes or so it takes to explain Ricardo. Krugman (1996) responds to these in three dimensions, i.e., first, it is fashionable to do so; since free trade and its underlying notion of comparative advantage are iconic among economists, attacking the icon seems to be fashionable; second, it is harder to understand than most people think, since it requires one to know how competitive market works and what is general equilibrium; third, the refusal of mathematical way of presentation and thinking, as most humanists who feel their territory is invaded by aliens armed with equations.

While the points that Krugman's made there are correct, on the concept of comparative advantage itself, he does not go beyond introducing it again in a very simple form. Rather he just unravels the culture and emotional or political factors

that nourish the refusal by intellectuals.

There are, however, serious and professional criticisms. But most criticisms in this sort attack on the assumptions rather than the logic of the theory itself, hence much of the negation is established based on the validity of theory, rather than the paradoxical nature of the theory.

For instance, Fletcher (2008) summaries the validity of the assumptions of the theory, and questions the applicability of the theory given the limited validity of these assumptions. There are seven assumptions questioned.

Assumption 1: There are no externalities. The author uses transboundary pollution as an example of negative externality and vaguely points out that pollution haven and global environment damage may occur if free trade is present. The second example is the wipeout of domestic technological spillover by heavy subsidized foreign industry.

Assumption 2: Nations trade only goods and services, not debt and assets. This may easily trigger endless debate on how money affects real economy. Because credit can be produced without any cost, the author concludes that this causes product market malfunction.The gigantic and growing amount of US trade deficit is an example of it.

Assumption 3: Factors of production are domestically mobile. The theory of comparative advantage assumes that free trade will reallocate factors of production from sectors with comparative disadvantage to sectors with comparative advantage. A counterexample is that labor can't move between industries – say, because of skill mismatches –workers out of sunset industries can not easily get into sunrise ones, but into unemployment.

Assumption 4: Factors of production are not internationally mobile. Harvard

professor Stephen Marglin points out that although one cannot move Portuguese vineyards to England,nor can England's lush sheep pastures survive in Portugal'sclimate, technology and capital move almost as easily across international borders as within a country (Marglin 2004). A natural corollary to international factor mobility is that comparative advantage will be replaced by absolute advantage, which governs competition between nations and unemployment will occur.

Assumption 5: Long-term growth is caused by short-term efficiency. The traditional theory of comparative advantage is a case of static analysis. But productivity changes are more crucial to overall growth. There are some researches on this and concerns are present. On strand of the literature focuses on the role of endogenous technological change, using Romer (1990)'s term. In particular, international knowledge spillover, and/or cross country differences in R&D efficiency result reversal of initial patterns of specialization (Grossman and Helpman, 1991). Grossman and Helpman allow for a wide range of resource endowments and sectoral adjustment within countries and trade still leads to convergence of growth rates across countries. If international diffusion of knowledge is not accompanied with trade simultaneously, then trade in goods can lead to a divergence of growth rates.

The second strand of the research is based on learning-by-doing or factor accumulation (Lucas (1988), Findlay 1970, Deardoff, 1974 etc.). Several measures of Revealed Comparative Advantage have been proposed and there is empirical evidence of trade and production pattern changes in the literature.

Indeed, some attribute the dynamic efficiency obtained by not following comparative advantage of a country is a major reason why nations that have openly rejected the theory as a guide to policy, like Japan and China, have been able to

succeed so brilliantly without it.

Assumption 6: There are no economies of scale. In the presence of scale economies, nations that reach large-scale production first can entrench themselves in industries simply because they were first. Economists and business leaders know that certain industries constitute global oligopolies that can reap exceptional profits and pay exceptional wages even in the face of cheap foreign labor.

Assumption 7: There is no cross-border investment. This assumption is related to assumption 3 discussed above. In fact, having observed that capital mobility may arise, Ricardo then argues why capital will not be mobile, as he writes:

> Experience, however, shows that the fancied or real insecurity of capital, when not under the immediate control of its owner, together with the natural disinclination which every man has to quit the country of his birth and connections, and entrust himself, with all his habits fixed, to a strange government and new laws, check the emigration of capital. These feelings, which I should be sorry to see weakened, induce most men of property to be satisfied with a low rate of profits in their own country, rather than seek a more advantageous employment for their wealth in foreign nations.

Yet this assumption has been invalidated by the recent tendency for global capital flows to exceed global trade flows by orders of magnitude.

The anti-globalization sentiment in the 1990s raised several challenges to free trade from a practitioner's perspective. For instance, fair trade as a precondition for free trade, adverse income distributional effects of trade etc., along with the old challenges such as factor and product market imperfections, are also related to

the critics of the predictions of comparative advantage theory. Bhagwati (1994) has a thorough discussion on challenges of free trade practice.

But none of these critics are central to the internal logic of either Ricardian or Heckscher-Ohlin model of comparative advantage. They are, in most cases, questions of the validity of the assumptions, hence the problem of applicability of comparative advantage theory and consequently whether free trade should be ardently advocated.

There is one exception, that is, the Pareto Paradox. Pareto was fascinated by Ricardo's model of comparative advantage and later in the Manual attempted to contest one of Ricardo's assertions. He showed that if two agents fully specialize in their products of comparative advantage, the world output of one of the products may decline below its autarky level, making one of the agent worse off. Chipman (1965) dubbed this as 'Pareto Paradox'. Pareto showed by a numerical example that the output of one of the commodities can decline if trade occurs and the larger output of the other commodity may not offer a sufficient compensation for this. In fact, Mill (1920) independently discovered this paradox and indeed, the condition for which trade is beneficial to both agents is called 'Mill's inequality'. Chipman (1965) argues that Pareto's criterion (or the absence of Pareto's Paradox) is neither necessary nor sufficient if elasticity of substitution is not specified. One notices that Pareto's criterion relies on the choice of functional forms of utility functions, and obviously the paradox does not carry over to the Heckscher-Ohlin model. The Pareto's Paradox then does not constitute a serious challenge to the comparative advantage theory.

We, however, question the logical paradigm of the comparative advantage theory, or the notion of comparative advantage itself, on what is the true meaning in

full scale, what implications does it have, and if it is logically incorrect in general, under what circumstances will its basic predictions hold?

To do so, we stick to all assumptions in the original H-O model such as perfect competition, international immobility of factors, domestic mobility of factors, 2 factor-2 product- 2 country benchmark case etc. This paper is not concern with applicability of the model by investigating validity of its assumptions, rather, we also rely on the assumptions that abstract away the irrelevant (to the core of the idea of comparative advantage) factors affecting the analysis.

The motivation is the following. If comparative advantage across nations can bring benefit through free trade, what if two previously separated countries (with relative endowment and different comparative advantages) merge into one? Note that before and post merger, we assume free trade between the two nations (regions after merger), as well as with the third country (for simplicity, just assume it is the rest of the world). The question is essentially whether the two former countries, now regions, average out their factor content of production according to the newly formed country's average comparative advantage against the ROW, or continue to produce what they produced before merger? Obviously any answer to this is paradoxical if we stick to the fundamentalist's concept and model of comparative advantage. If one answers yes, this is to say that before merger specialization is not optimal, and if answers no, this is in conflict with the original concept of comparative advantage which says a country should specialize on the sector which uses factor that rich in there. Similarly, if a country is divided into two or more parts, the question is then whether the production should be reorganized simply because the political regime has been changed?

This is comparable to the discovery of law of free fall by Galileo. In Aristotle's

De Caelo, he asserts that larger and/or heavy bodies falls quicker. In the historical experiment Galileo made on the famous leaning tower of Pisa, it was shown that all bodies fall at the same speed. But it is not necessary at all to conduct the real experiments to illustrate the law. Galileo proved that Aristotle's thesis leads to contradiction by thought (gedanken) experiment rather than real experiment, i.e, if heavy body falls faster than a light body, what happens when the two bodies are combined?

Similar to Aristotle's theory, the comparative advantage theory predicts full specialization (Ricardo) or first best through specializing on the only commodity that intensively uses the factor a nation is richly endowed (H-O). But consider the situation, say, China and Hong Kong, before the 1997 hand over, the two were under different political sovereignty so they should trade each other and with the ROW according to their factor endowment comparative advantages. China is relatively rich in labor and Hong Kong is abundant of capital. Capital was almost perfect mobile across border. Labor was somehow restricted but entrepreneurs in Hong Kong were free to setup firms and hire cheap labor at the neighboring city – Shenzhen. But after 1997, Hong Kong became part of China officially. The original comparative advantage theory tells us that either we should knock down the skyscrapers in Hong Kong and plant cotton or establish shoe factories according to the country's comparative advantage which is ridiculous, or stick to the before handover production pattern because Hong Kong is comparatively abundant in capital but contradict the theory itself.

In a realistic situation, even without consideration of productivity change or convergence/divergence dynamics, whether to carry out industrialization in a labor abundant country is of vital importance, both theoretically and practically. In

the framework of comparative advantage theory, we argue that industrialization (in some regions) in a labor abundant country can be beneficial to the country because it may gain from paying less for import of capital-intensive products from the capital abundant country. The tradeoff is that gains from self-sufficiency the product that requires more factor that the country lacks (in relative terms) through savings from import outweigh the loss from not specializing (to the extreme case as Ricardo or H-O subscribe) on the product that requires the factor rich in this country in average.

In this sense, the traditional comparative advantage theory and factor endowment trade theory are normative rather than positive theories. They characterize the first best outcomes but fail to identify the equilibrium if a country is not restricted to a single region only. Without the plausibility of regionalization/industrialization within a country, the equilibrium trade coincides with the first best. However, when regionalization is allowed, world output (welfare) may be higher.

The logical paradox inherent to this problem is the concept of comparative advantage itself by its definition. If specialization and free trade are to be ardently advocated when comparative advantage is present, what is the logic for this seemingly straightforward statement? Put it in another way, why would a country specialize on the sector that uses the factor it is rich in, even if it comes at a high opportunity cost? If on the other hand, the country does not specialize as the way it "ought" to be, why should we care about comparative advantage at all?

This paper is, to the best of our knowledge, the first to address this paradox and present it using the standard H-O model and traditional treatment. It can be

shown, that with domestic regionalization, trade is beneficial to a nation, relative to the case of original H-O model. Under some conditions, world welfare is also higher relative to that in the original H-O model. This is because with domestic strategic regionalization, terms of trade are improved in the home country. In the original H-O story, although trade can be gainful to both countries, one country may end up paying too much labor intensive product in exchange for capital intensive product, or vice versa. This potential distortion on world prices (terms of trade) can be improved if the labor intensive country can foster a region with higher capital/labor ratio and start some sort of industrialization. Hence total world output may increase as a result.

We argue at the very least that the textbook comparative advantage and Ricardian/H-O models are normative rather than positive theories (which is commonly labeled and perceived). The concept and theory of comparative advantage deserve a serious treatment. There are many dimensions one can go, for example, the degree of optimal internal specialization in a multi-factor, multi-product model.

This paper also gives a justification of why governments in many developing countries, presumably relatively abundant in labor but scarce in capital, promote industrialization at a high cost. Special interest lobby models obviously cannot explain such widespread policy practice.

The rest of this paper is organized as follows: Section 2 introduces the basic model and the paradox itself; Section 3 concludes the model.

2. The Model

2.1. Model Setup

We work on the two-factor, two sector, two-country ($2\times2\times2$) framework which is regarded as the fundamental general equilibrium framework. The two countries are called Home and Foreign.

1. Home country is endowed with fixed amount of two homogenous factors, labor and capital. Denote the endowments of labor and capital by L^H and K^H, respectively. Foreign is also endowed with two factors, with fixed amount of L^H and K^H for labor and capital respectively. Without loss of generality, assume also that Home is relatively more labor abundant and Foreign is relatively more capital abundant, i.e., $\frac{L^H}{K^H} > \frac{L^F}{K^F}$.

2. Two sectors produce two homogeneous goods, labeled goods A and B. The production function of sector i, $i = 1, 2$, is

$$Q_i = F_i\left(K_i,\, L_i\right),$$

where Q_i is the output, and K_i and L_i are respectively the amounts of capital and labor employed in the sector. Each production function is increasing, concave, linearly homogeneous, and differentiable. Both factors are necessary in production. Two countries have the same technology and the production function is the same for each good in the two countries.

3. Consumers in the two countries have same preference.

4. The factors labor and capital are freely mobile between the two sectors in a country, however they are not mobile internationally, nor across regions within a

country is domestic regionalization takes place.

5. The market in either country is completely competitive. All the products will be consumed and there is no wasteful disposal.

Traditional Ricardian and Heckscher-Ohlin theories claim that each country produces according to their comparative advantage will make them better off.

In this paper, labor abundant country, or Home, is divided into two parts: I for inland and S for Shanghai.

2.2. The Original Heckscher-Ohlin Benchmark

First, we introduce some notations. Throughout, we use˜to denote the case of original H-O benchmark model. Let $\widetilde{X}_i^l$ denote the output of goods $i(i = A, B)$ in country $l(l = F, H)$. Denote $\widetilde{X}_A$ and $\widetilde{X}_B$ the world total output of goods A and B respectively. Let $\widetilde{C}_i^l$ be the consumption of goods $i(i = A, B)$ in country $l(l = F, H)$. Let K and L be the world total capital and labor respectively. Given the homothetic preferences and common goods prices, the consumption of good i in country l can be expressed as $\widetilde{C}_i^l = \widetilde{s}^l \widetilde{X}_i$, where $\widetilde{s}^l$ is l's share of world income. Denote by $c_i\left(w^l, r^l\right)$ the unit cost function in sector i under the factor prices in country l, that is

$$c_i\left(w^l, r^l\right) = \min_{L_i, K_i \geq 0} \left\{ w^l L_i + r^l K_i | F_i\left(K_i, L_i\right) \geqslant 1 \right\}.$$

Following the properties of $F_i\left(\cdot\right)$, $c_i\left(\cdot\right)$ will also be homogenous of degree 1 and twice continuously differentiable. Denote by $a_{ji}\left(w^l, r^l\right)$ the unit demand of factor $j = K, L$ in the production of good i in country k. Applying Shephard's lemma,

we have

$$a_{Li}\left(w^l, r^l\right) = \frac{\partial c_i\left(w^l, r^l\right)}{\partial w^l}; \quad a_{Ki}\left(w^l, r^l\right) = \frac{\partial c_i\left(w^l, r^l\right)}{\partial r^l}.$$

In the original H-O framework, the factor market clearing condition for Home country in standard textbooks is,

$$\begin{bmatrix} \widetilde{X}_A^H - \widetilde{C}_A^H \\ \widetilde{X}_B^H - \widetilde{C}_B^H \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} L^H \\ K^H \end{bmatrix} - \mathbf{A}^{-1} \cdot \widetilde{s}^H \begin{bmatrix} L \\ K \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} L^H - \widetilde{s}^H L \\ K^H - \widetilde{s}^H K \end{bmatrix} \tag{1}$$

where $\mathbf{A}^{-1}$ is technology matrix, $\widetilde{s}^H$ is Home's share of world income, and

$$\mathbf{A} = \begin{bmatrix} a_{LA}(w) & a_{LB}(w) \\ a_{KA}(w) & a_{KB}(w) \end{bmatrix}$$

and

$$\widetilde{s}^H = \frac{\widetilde{w} \cdot L^H + \widetilde{r} \cdot K^H}{\widetilde{w}\left(L^H + L^F\right) + \widetilde{r}\left(K^H + K^F\right)}.$$

Likewise, the factor market clearing conditions for Foreign country is,

$$\begin{bmatrix} \widetilde{X}_A^F - \widetilde{C}_A^F \\ \widetilde{X}_B^F - \widetilde{C}_B^F \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} L^F \\ K^F \end{bmatrix} - \mathbf{A}^{-1} \cdot \widetilde{s}^F \begin{bmatrix} L \\ K \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} L^F - \widetilde{s}^F L \\ K^F - \widetilde{s}^F K \end{bmatrix} \tag{2}$$

where $\widetilde{s}^F$ is Foreign's share of world income and

$$\widetilde{s}^F = \frac{\widetilde{w} \cdot L^H + \widetilde{r} \cdot K^H}{\widetilde{w}L + \widetilde{r}K}.$$

In equilibrium, all the factors are fully utilized, that is,

$$a_{LA}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)\left(\widetilde{X}_A^H + \widetilde{X}_A^F\right) + a_{LB}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)\left(\widetilde{X}_B^H + \widetilde{X}_B^F\right) = L^H + K^F,$$

and

$$a_{KA}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)\left(\widetilde{X}_A^H+\widetilde{X}_A^F\right)+a_{KB}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)\left(\widetilde{X}_B^H+\widetilde{X}_B^F\right)=K^H+K^F.$$

No disposal implies that all productions are consumed, or,

$$\widetilde{X}_A^F+\widetilde{X}_A^H=\widetilde{C}_A^H+\widetilde{C}_A^F$$

and

$$\widetilde{X}_B^F+\widetilde{X}_B^H=\widetilde{C}_B^H+\widetilde{C}_B^F.$$

Equilibrium world total outputs of goods B and goods A in the original H-O benchmark model are,

$$\widetilde{X}_B^H+\widetilde{X}_B^F=\frac{L\widetilde{a}_{KA}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)-K\widetilde{a}_{LA}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)}{\widetilde{a}_{LB}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)\cdot\widetilde{a}_{KA}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)-\widetilde{a}_{KB}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)\cdot\widetilde{a}_{LA}\left(\frac{\widetilde{w}}{\widetilde{r}}\right)}, \tag{3}$$

and

$$\widetilde{X}_A^H+\widetilde{X}_A^F=\frac{\widetilde{a}_{KB}L-K\cdot\widetilde{a}_{LB}}{\widetilde{a}_{LA}\cdot\widetilde{a}_{KB}-\widetilde{a}_{KA}\cdot\widetilde{a}_{LB}}, \tag{4}$$

where $\widetilde{w}$ and $\widetilde{r}$ are the equilibrium returns for labor and capital in the benchmark H-O model.

2.3. Domestic Regionalization

In the case where Home country is divided into two regions with different ratio of endowment, and we restrict factor mobility across regions. Similarly, let $\overline{X}_i^l$ denote the output of goods $i(i=A,B)$ in area $l(l=S,I,F)$. Factor endowment in each country is the same with the benchmark, that is, $L^I+L^S=L^H$, $K^I+K^S=K^H$. Denote by $\overline{C}_i^l$ the consumption of goods $i(i=A,B)$ in area $l(l=S,I,F)$. Let η and λ be the proportions of labor and capital in Home country that are

distributed to area S. Denote $\overline{X}_A$ and $\overline{X}_B$ world total output of goods A and B respectively. Throughout, we use to denote the case of domestic regionalization. Our first purpose is to obtain the trade pattern of Foreign country and the two areas in Home country. Given homothetic preferences and common goods prices, the consumption of good i in country or area l can be expressed as $\overline{C}_i^l = s^l \overline{X}_i$, where s^l is l's share of the world income. Then from the factor market clearing conditions in area S, we have,

$$\begin{bmatrix} \overline{X}_A^S - \overline{C}_A^S \\ \overline{X}_B^S - \overline{C}_B^S \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \eta L^H \\ \lambda K^H \end{bmatrix} - \mathbf{A}^{-1} \cdot m^S \begin{bmatrix} L \\ K \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \eta L^H - m^S L \\ \lambda K^H - m^S K \end{bmatrix} \quad (5)$$

where m^S is S's share of world income and

$$m^S = \frac{\overline{w} \cdot \eta L^H + \overline{r} \cdot \lambda K^H}{\overline{w} L + \overline{r} K}.$$

From the factor market clearing conditions in area I, we have

$$\begin{aligned} \begin{bmatrix} \overline{X}_A^I - \overline{C}_A^I \\ \overline{X}_B^I - \overline{C}_B^I \end{bmatrix} &= \mathbf{A}^{-1} \begin{bmatrix} (1-\eta) L^H \\ (1-\lambda) K^H \end{bmatrix} - \mathbf{A}^{-1} \cdot m^I \begin{bmatrix} L \\ K \end{bmatrix} \quad (6) \\ &= \mathbf{A}^{-1} \begin{bmatrix} (1-\eta) L^H - m^I L \\ (1-\lambda) K^H - m^I K \end{bmatrix} \end{aligned}$$

where m^I is I's share of world income and

$$m^I = \frac{\overline{w} \cdot (1-\eta) L^H + \overline{r} \cdot (1-\lambda) K^H}{\overline{w} \cdot (L^H + L^F) + \overline{r} \cdot (K^H + K^F)}.$$

Likewise, from the factor market clearing conditions in Foreign country, we have

$$\begin{bmatrix} \overline{X}_A^F - \overline{C}_A^F \\ \overline{X}_B^F - \overline{C}_B^F \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} L^F \\ K^F \end{bmatrix} - \mathbf{A}^{-1} m^F \begin{bmatrix} L \\ K \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} L^F - m^F L \\ K^F - m^F K \end{bmatrix} \tag{7}$$

where m^F is F's share of world income and

$$m^F = \frac{\overline{w} \cdot L^F + \overline{r} \cdot K^F}{\overline{w} L + \overline{r} K}.$$

Proposition 1 *(Modified Heckscher-Ohlin Theorem) Region I is labor insentive goods exporter, and region S and country F are both labor-insentive goods importer and capital-insentive goods exporter, if and only if*

$$\frac{a_{KA} + a_{KB}}{a_{LA} + a_{LB}} < \frac{-\overline{w} L^H K \eta + \overline{w} K^H L \lambda + \overline{w} L^H K^F - \overline{r} K^H K^F}{\overline{r} L^H K \eta - \overline{r} K^H L \lambda + \overline{r} L^F K^H - \overline{r} L^H K^F}$$

Proof. Expand trade pattern equations (5), (6), and (7) to

$$\begin{bmatrix} \overline{X}_A^S - \overline{C}_A^S \\ \overline{X}_B^S - \overline{C}_B^S \end{bmatrix} = \frac{1}{|\mathbf{A}|} \begin{bmatrix} a_{KB}[\eta L^H - m^S L] - a_{LB}[\lambda K^H - m^S K] \\ -a_{KA}[\eta L^H - m^S L] + a_{LA}[\lambda K^H - m^S K] \end{bmatrix},$$

$$\begin{bmatrix} \overline{X}_A^I - \overline{C}_A^I \\ \overline{X}_B^I - \overline{C}_B^I \end{bmatrix} = \frac{1}{|\mathbf{A}|} \begin{bmatrix} a_{KB}[(1-\eta) L^H - m^I L] - a_{LB}[(1-\lambda) K^H - m^I K] \\ -a_{KA}[(1-\eta) L^H - m^I L] + a_{LA}[(1-\lambda) K^H - m^I K] \end{bmatrix},$$

and

$$\begin{bmatrix} \overline{X}_A^F - \overline{C}_A^F \\ \overline{X}_B^F - \overline{C}_B^F \end{bmatrix} = \frac{1}{|\mathbf{A}|} \begin{bmatrix} a_{KB}[L^F - m^F L] - a_{LB}[K^F - m^F K] \\ -a_{KA}[L^F - m^F L] + a_{LA}[K^F - m^F K] \end{bmatrix}.$$

Because goods A is more labor intensive than goods B, then we have $\frac{a_{LA}}{a_{KA}} > \frac{a_{LB}}{a_{KB}}$, then the sign of $|A|$ will be positive. Note that area S is a capital abundant area, and area I is a labor abundant area in Home country. We want find condition(s) for the followings to hold:

$\overline{X}_A^S - \overline{C}_A^S < 0$
$\overline{X}_B^S - \overline{C}_B^S > 0$
$\overline{X}_A^I - \overline{C}_A^I > 0$
$\overline{X}_B^I - \overline{C}_B^I < 0$
$\overline{X}_A^F - \overline{C}_A^F < 0$
$\overline{X}_B^F - \overline{C}_B^F > 0.$

Simple algebra yields that the condition for the above six inequalities to hold is: $(a_{KA} + a_{KB})[2\eta L^H + L^F - L^H + (m^I - m^S - m^F)L] < (a_{LA} + a_{LB})[2\lambda K^H + K^F - K^H + (m^I - m^S - m^F)K]$

Rearranging terms, the condition can be written as,

$$\frac{a_{KA} + a_{KB}}{a_{LA} + a_{LB}} < \frac{-\overline{w}L^H K\eta + \overline{w}K^H L\lambda + \overline{w}L^H K^F - \overline{r}K^H K^F}{\overline{r}L^H K\eta - \overline{r}K^H L\lambda + \overline{r}L^F K^H - \overline{r}L^H K^F}.$$

■

Proposition 1 implies that if η is small enough and/or λ is large enough, i.e., when labor share is low enough and capital share is high enough in Shanghai, I will be a net exporter of labor insentive goods to S and F. and S and F will export capital intensive goods to I. The followings can be then established,

$$\overline{C}_A^S + \overline{C}_A^F = \overline{X}_A^S + \overline{X}_A^F + \left(\overline{X}_A^I - \overline{C}_A^I\right),$$

and

$$\overline{C}_B^I = \left(\overline{X}_B^S - \overline{C}_B^S\right) + \left(\overline{X}_B^F - \overline{C}_B^F\right) + \overline{X}_A^F.$$

In the equilibrium of trade among country Foreign and area S and I, there is an important condition that links unit production cost, world total output and world total endowments together, that is, all the factors in the world will be fully utilized by the two sectors, i.e.,

$$\overline{a}_{LA}\left(\frac{\overline{w}}{\overline{r}}\right)\cdot\overline{X}_A + \overline{a}_{LB}\left(\frac{\overline{w}}{\overline{r}}\right)\cdot\overline{X}_B = L^H + L^F$$

$$\overline{a}_{KA}\left(\frac{\overline{w}}{\overline{r}}\right)\cdot\overline{X}_A + \overline{a}_{KB}\left(\frac{\overline{w}}{\overline{r}}\right)\cdot\overline{X}_B = K^H + K^F$$

$$\overline{X}_A = \overline{X}_A^S + \overline{X}_A^I + \overline{X}_A^F$$

$$\overline{X}_B = \overline{X}_B^S + \overline{X}_B^I + \overline{X}_B^F$$

$$\overline{a}_{LA}\left(\frac{\overline{w}}{\overline{r}}\right)\left(\overline{X}_A^S + \overline{X}_A^I + \overline{X}_A^F\right) + \overline{a}_{LB}\left(\frac{\overline{w}}{\overline{r}}\right)\left(\overline{X}_B^S + \overline{X}_B^I + \overline{X}_B^F\right) = \sum L^l \quad (8)$$

and

$$\overline{a}_{KA}\left(\frac{\overline{w}}{\overline{r}}\right)\left(\overline{X}_A^S + \overline{X}_A^I + \overline{X}_A^F\right) + \overline{a}_{KB}\left(\frac{\overline{w}}{\overline{r}}\right)\left(\overline{X}_B^S + \overline{X}_B^I + \overline{X}_B^F\right) = \sum K^l \quad (9)$$

where $a_{LA}(\cdot)$, $a_{LB}(\cdot)$, $a_{KA}(\cdot)$, and $a_{KB}(\cdot)$ are the unit cost function of the production of goods A and B as functions of the ratio of the factor returns. Total world output of the two goods in equilibrium can be obtained as,

$$\begin{aligned} \overline{X}_A^S + \overline{X}_A^I + \overline{X}_A^F &= \overline{C}_A^S + \overline{C}_A^I + \overline{C}_A^F \\ &= \frac{\left(L^H + L^F\right)\overline{a}_{KA}\left(\frac{\overline{w}}{\overline{r}}\right) - \left(K^H + K^F\right)\overline{a}_{LA}\left(\frac{\overline{w}}{\overline{r}}\right)}{\overline{a}_{LB}\cdot\overline{a}_{KA} - \overline{a}_{KB}\cdot\overline{a}_{LA}}, \end{aligned} \quad (10)$$

and

$$\begin{aligned} \overline{X}_B^S + \overline{X}_B^I + \overline{X}_B^F &= \overline{C}_B^S + \overline{C}_B^I + \overline{C}_B^F \\ &= \frac{\left(L^H + K^F\right) \cdot \overline{a}_{KB} - \left(K^H + K^F\right) \cdot \overline{a}_{LB}}{\overline{a}_{LA} \cdot \overline{a}_{KB} - \overline{a}_{KA} \cdot \overline{a}_{LB}}. \end{aligned} \tag{11}$$

Under perfect competition, firms produce positive outputs and earn zero profit, which means that unit costs are equal to the price of the goods in the markets. Therefore the world equalibrium price are determined by the following equations,

$$\overline{P}_A^* = \overline{w} \cdot \overline{a}_{LA}\left(\frac{\overline{w}}{\overline{r}}\right) + \overline{r} \cdot \overline{a}_{KA}\left(\frac{\overline{w}}{\overline{r}}\right), \tag{12}$$

and

$$\overline{P}_B^* = \overline{w} \cdot \overline{a}_{LB}\left(\frac{\overline{w}}{\overline{r}}\right) + \overline{r} \cdot \overline{a}_{KB}\left(\frac{\overline{w}}{\overline{r}}\right), \tag{13}$$

where $\overline{w}$ and $\overline{r}$ are the equilibium returns to labor and capital, respectively.

2.4. Factor Price Equalization

Since the production function is linearly homogeneous, then the output-labor ratio of sector i can be given by function $f_i(k_i) \equiv F_i(k_i, 1)$, where $k_i \equiv \frac{K_i}{L_i}$ is the capital-labor ratio. Thus, $Q_i = L_i f_i(k_i) \equiv L_i F_i(K_i/L_i, 1)$. The marginal products of capital and labor are respectively equal to

$$F_{iK}(K_i,\, L_i) \equiv \frac{\partial F_i(K_i, L_i)}{\partial K_i} = \frac{\mathrm{d}f_i}{\mathrm{d}k_i} \equiv f_i'(k_i) \tag{14}$$

and

$$F_{iL}(K_i,\, L_i) \equiv \frac{\partial F_i(K_i, L_i)}{\partial L_i} = f_i(k_i) - k_i f_i'(k_i) \tag{15}$$

Proposition 2 *The ratio of rental over wage is lower in domestic regionalization relative to the original H-O benchmark iff the marginal rate of transformation is lower, or*

$$iff\ \frac{\widetilde{F}^H_{BK}}{\widetilde{F}^H_{BL}} > \frac{\overline{F}^S_{BK}}{\overline{F}^S_{BL}}\ ,\ then \frac{\widetilde{r}}{\widetilde{w}} > \frac{\overline{r}}{\overline{w}}.$$

Proof.

Remark 3 *In the original H-O benchmark model, equilibrium returns of factors equal to the marginal value of production, i.e.,*

$$\widetilde{r} = \widetilde{P}^*_A \cdot f^{H\prime}_A|_{\mathbf{X}=\widetilde{\mathbf{X}}} = \widetilde{P}^*_B \cdot f^{H\prime}_B|_{\mathbf{X}=\widetilde{\mathbf{X}}}$$

and

$$\begin{aligned} \widetilde{w} &= \widetilde{P}^*_A \left(f^H_A - k^H_A \cdot f^{H\prime}_A\right)|_{\mathbf{X}=\widetilde{\mathbf{X}}} \\ &= \widetilde{P}^*_B \left(f^H_B - k^H_B \cdot f^{H\prime}_B\right)|_{\mathbf{X}=\widetilde{\mathbf{X}}}, \end{aligned}$$

where $k^H_A = \frac{K^H_A}{L^H_A}$, *and* $k^F_A = \frac{K^F_A}{L^F_A}$.

If on the other hand, the Home country implements domestic regionalization specified in this paper, then in equilibrium the returns to factors which are equal to the marginal values of production, can be written as,

$$\overline{r} = \overline{P}^*_A \cdot f^{S\prime}_A|_{\mathbf{X}=\overline{\mathbf{X}}} = \overline{P}^*_B \cdot f^{S\prime}_B|_{\mathbf{X}=\overline{\mathbf{X}}}$$

$$\overline{w} = \overline{P}^*_A \left(f^S_A - k^S_A \cdot f^{S\prime}_A\right)|_{\mathbf{X}=\overline{\mathbf{X}}} = \overline{P}^*_B \left(f^S_B - k^S_B \cdot f^{S\prime}_B\right)|_{\mathbf{X}=\overline{\mathbf{X}}}.$$

Some simple algebra yields

$$\frac{\widetilde{r}}{\widetilde{w}} = \frac{f_B^{H\prime}|_{\mathbf{X}=\widetilde{\mathbf{X}}}}{f_B^H|_{\mathbf{X}=\widetilde{\mathbf{X}}} - k_B^H \cdot f_B^{H\prime}|_{\mathbf{X}=\widetilde{\mathbf{X}}}},$$

and

$$\frac{\overline{r}}{\overline{w}} = \frac{f_B^{S\prime}|_{\mathbf{X}=\overline{\mathbf{X}}}}{f_B^S|_{\mathbf{X}=\overline{\mathbf{X}}} - k_B^S \cdot f_B^{S\prime}|_{\mathbf{X}=\overline{\mathbf{X}}}}.$$

Because of domestic regionalization, we have $f_B^{H\prime}|_{\mathbf{X}=\widetilde{\mathbf{X}}} = f_B^{S\prime}|_{\mathbf{X}=\overline{\mathbf{X}}}$. *It is also obvious that* $k_B^S > k_B^H$. *But the sign of* $\left[f_B^H|_{\mathbf{X}=\widetilde{\mathbf{X}}} - f_B^S|_{\mathbf{X}=\overline{\mathbf{X}}}\right]$ *is indeterminate. Thus the real returns to factors depends on how production possibility frontier changes upon regionalization.*

■

2.5. The Paradox

In this subsection, the total output of the two goods and terms of trade in the two situations will be compared. Also based on Proposition 1 and 2, we want to obtain some useful conclusions. Before reaching our conclusion, the difference of output in different situations should be compared. It is expected that the difference would be a function of the factor endowment and unit production cost function. The difference of output of B and A in the original H-O model and in the present model in which domestic regionalization takes place, can be written as,

$$\begin{aligned} & \left(\overline{X}_B^S + \overline{X}_B^I + \overline{X}_B^F\right) - \left(\widetilde{X}_B^H + \widetilde{X}_B^F\right) \qquad (16) \\ = \; & \frac{L\overline{a}_{KA} - K\overline{a}_{LA}}{\overline{a}_{LA}\cdot\overline{a}_{KB} - \overline{a}_{KA}\cdot\overline{a}_{LB}} - \frac{\left(L^H + L^F\right)\widetilde{a}_{KA} - K\widetilde{a}_{LA}}{\widetilde{a}_{LA}\cdot\widetilde{a}_{KB} - \widetilde{a}_{KA}\cdot\widetilde{a}_{LB}} \end{aligned}$$

and

$$\begin{aligned} & \left(\overline{X}_A^S + \overline{X}_A^I + \overline{X}_A^F\right) - \left(\widetilde{X}_A^H + \widetilde{X}_A^F\right) \\ = \quad & \frac{L\overline{a}_{KB} - K\overline{a}_{LB}}{\overline{a}_{LA}\cdot\overline{a}_{KB} - \overline{a}_{KA}\cdot\overline{a}_{LB}} - \frac{L\widetilde{a}_{KB} - K\widetilde{a}_{LB}}{\widetilde{a}_{LA}\cdot\widetilde{a}_{KB} - \widetilde{a}_{KA}\cdot\widetilde{a}_{LB}} \end{aligned} \tag{17}$$

where $a_{LA}(\cdot)$ and $a_{LB}(\cdot)$ are increasing functions of $\frac{r}{w}$, and $a_{KA}(\cdot)$ and $a_{KB}(\cdot)$ are decreasing functions of $\frac{r}{w}$. This is because the higher the ratio of rental to wage, the lower relative price of labor and consequently more labor will be employed in the production. By the same argument, there will be less capital employed.

Proposition 4 ***Corollary 5*** *(Terms of trade) If in equilibrium, $\frac{\widetilde{r}}{\widetilde{w}} > \frac{\overline{r}}{\overline{w}}$, then by Stolper-Samuelson Theorem, we have $\frac{\widetilde{P}_A^*}{\widetilde{P}_B^*} < \frac{\overline{P}_A^*}{\overline{P}_B^*}$, i.e., the terms of trade for region I is improved, and terms of trade for Foreign is worsened.*

The graphical interpretations are given in Figures 1 and 2.

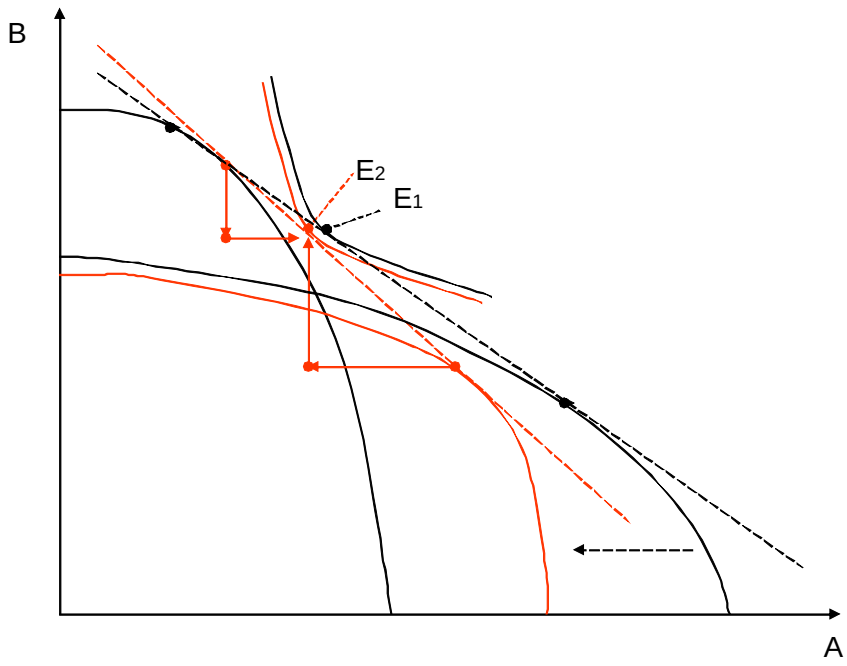

Figure 1. Benchmark H-O model with and without domestic regionalization

Figure 1 illustrates that when domestic regionalization at home takes place, the aggregate production possibility frontier shrinks since factors are not mobile across Shanghai and Inland (thus production can not be organized in the most efficient form nation wise). Then if we treat the post regionalization Home as a single region/country, obviously according to textbook Heckscher-Ohlin model, the trade equilibrium changes from E_1 to E_2 which is a Pareto inferior movement. However this is against the purpose of domestic regionalization.

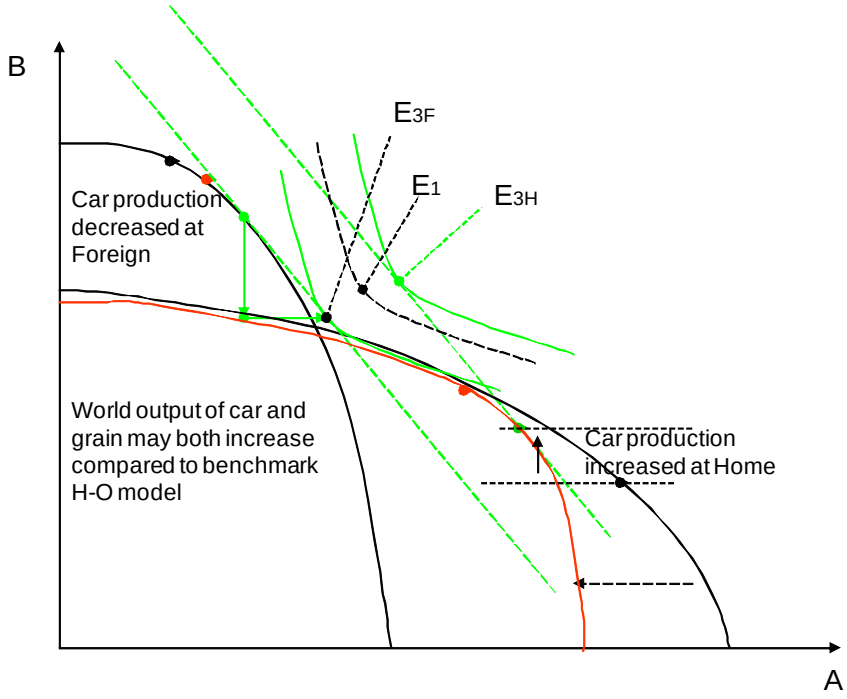

Figure 2. Equilibrium under domestic regionalization

Figure 2 illustrates the terms of trade improvement effect resulted by domestic regionalization. It is shown that when terms of trade improves, the Home country as a whole is settled at E_{3H} which is further northeast to E_1, hence representing a higher utility. The welfare of Foreign country decreases from E_1 to E_{3F}, however it is still gainful to trade with the Home country. World total welfare change is unclear graphically.

3. Conclusion

This paper studies the case where a country (WLG call it home) may intentionally divide itself into two regions with different factor ratios in order to gain

more from trade. While other critics of comparative advantage theory are centered on the validity of the assumptions and hence applicability of the theory, or the dynamic variant of H-O framework, the current paper sticks to the original static H-O framework with all assumption obeyed.

It is striking that under some mild conditions, home country can benefit more from trade through domestic regionalization which is essentially specializing on the sector that it does not have a comparative advantage. It is even more surprising that world welfare may be higher under domestic regionalization which is apparently "inefficient" from traditional H-O point of view.

When domestic regionalization is introduced, the labor abundant (capital abundant) home country import less capital-intensive product in equilibrium, and the capital abundant country produces less capital-intensive product compared to original H-O benchmark model. Nevertheless, world output on both sectors may increase. Together with the fact that the home country enjoys a higher welfare, this model provides a justification for the movement of "self-sufficiency" programs or "import substitution" industries in many developing countries.

We argue that the reason is that although boosting up industries not in a nation's comparative advantage may be wrong at the first glance, a country does so can improve her terms of trade that more than offset the cost of inefficiency in production as H-O predicts.

We then claim that this constitutes a paradox to the notion of comparative advantage. On one hand, if countries do not specialize and hence no need to trade, world welfare is low and there is room for Pareto improvement through trade; on the other hand, specializing in the way that plain-vanila H-O model suggests may not be in the interest of a country, and even of the world. The

model presented in this paper can be seen as a counter-example to illustrate the paradoxical situation of the traditional theory of comparative advantage.

References

Bhagwati, Jagdish, (1994), Free Trade: Old and New Challenges, *Economic Journal*, (104):231-246

Chipman, J.S. (1965), A Survey of the Theory of International Trade: Part 1, The Classical Theory,' *Econometrica*, 33(3): 477-519.

Deardorff, A., (1974), Factor proportions and comparative advantage in the long-run: comment. *Journalof Political Economy* 82 (4), 829–833.

Findlay, R., 1970. Factor proportions and comparative advantage in the long run. *Journal of Political Economy* 78 (1), 27–34.

Fletcher, Ian (2008), Fatal Flows in the Theory of Comparative Advantage, AmericanEconomicAlert.org

Grossman, G., Helpman, E., (1991), Innovation and Growth in the Global Economy. MIT Press, Cambridge, MA.

Krugman, Paul, (1996), Ricardo's Difficult Idea (paper for Manchester conference on free trade, March 1996, web.mit.edu/krugman/www/ricardo.htm)

Lucas, Robert E., (1988), On the mechanics of economic development, *Journal of Monetary Economics* 22, 3-42.

Marglin, Stephen, (2004), Outsourcing Common Sense, Newsletter of The Programmers Guild, August 2004.

Mill, John Stuart, (1920), Principles of Political Economy, 7th ed., London: Longman, Green and Co.

Pareto, V. (1906), Manuale dieconomia politica, Milan: Societa Editrica Libraria.

Romer, Paul M., 1990, Endogenous technological change, *Journal of Political Economy* 98, S71-S102.